A Synopsis

of the proposed thesis entitled**,**

“Generalised result of zero divisor graphs”

Submitted to

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1.Abstract:

In this research work we study generalised results of zero divisor graphs.We also study new zero divisor graphs of ring, module. Also with the help of zero divisor graph,we will discussing the properties of graph (connectivity,diameter,radius,completness,etc).We will check all this properties of graph to module.

2.Introduction

The graph theory is helps to find illustrative ways of seeing the relationship of its elements.zero divisor graphs is an overlaps of ring theory and graph theory which specifically illustrate the relationship between a ring zero divisor.when some ring have complicated behaviour we cannot easily understand it. Precisely the graph theory and zero divisor graph are widely useful for the theoretic properties of rings.for each commutative ring we associate a (simple) graph $\Gamma(R)$.we investigate the rings theoretic properties of R and graph theoretic properties of $\Gamma(R)$.we associate a graph in a natural way with the zero divisor of commutative ring we endeavour of characterise various attributes of the graph ,including connectivity, diameter and symmetry.The zero divisor relation is reflexive, symmetric in a ring but not transitive .

The zero divisor graph relation lends to an immediate identification with a simple graph .The zero divisor graph seems to extract certain essential information relative to zero divisor. During this work we will study the properties of zero divisor graphs. First we will done some Ring Theory and then some Graph theory. During this project we use the Anderson and Livingston graphs. Then we will discuss the methods we used to come to our conclusions. Then we will discuss Our findings and

provide an analysis for the different types of behaviour of these graphs. We only looked at graphs for the integers modulo n. The Beck graph designate a vertex for each zero divisor of the ring, but the structure of these graphs are not very interesting since all elements would be adjacent to the element '0'. We can see more interesting structure if we were to eliminate the zero element from the graphs. These are the graphs that Anderson and Livingston focused on. Therefore, when we consider a graph for being a possible zero divisor graph,We treated the graph as if it is of Anderson and Livingston zero divisor graph. Though our topic called for us to find rings associated with given graphs, where possible, but looking at how these graph are constructed will help understand the relationships illustrated in a zero divisor graph.

3.Review of literature and development in the subject

The idea of zero divisor graph of commutative ring was introduced by I.Beck in 1988. Let R be ring and *Z(R)* be its set of zero divisor.we associate graph (simple) *(R)* to *Γ(R)* with vertices $Z(R)^* = Z(R) - \{0\}$ set of non zero element of ring R.for distinct x and y € $Z(R)^*$,The vertices x and y are adjacent if and only if *x*y=0* .we study the following definition of graphs and zero divisor graph which are useful for achievement of result.

3.1Definitions:

I. **Graph :** A graph is a pair G=(V,E) of set satisfying $E \leq [V]^2$.Thus element of E are 2-element subset of V and V∩E=φ The element of V are vertices and element of E are edges .

II. **Zero divisor** : a zero divisor is a element of is a element r in a ring R such that r.s=0 for some non zero s in R

III. **Complete graph** : A graph is complete if every vertex in the graph is adjacent to other vertex .

IV. **Complete Bipartite Graph** : A complete bipartite graph is graph whose vertex set can be subset portioned into two disjoint $u_i$and v_j such that each u_i is adjacent to every u_i but no two are adjacent and no two are adjacent .

V. **Zero divisor graph (I.Beck1988**) The zero divisor graph of a ring R is a simple graph whose set of vertices consist of all element of the ring with edge defined between a and b if and if *a*b=0* it will be denoted by *G(R).*

4.Aim and objectives of Research

- To study all graphs and basic definition of graph theory.
- To study different types of ring (commutative and non commutative).
- We define and study the zero divisor graphs, in particular the Anderson and Livingston zero divisor graphs.
- We will discuss how to create them and what methods we used to find pattern in these graphs.
- We will discussing the behaviour of the ring as well as module.
- To study different types of zero divisor graph of ring Z_n ,when
- $n=p\ q$
- $n=p\ q\ r$
- $n=p^2q$
- $n=p^m\ q$
- We examine Automorphism group of the graph, provide a complete characterization for ring.
- Obtain zero divisor graphs of module by using new definitions and new results.

- We will look at the generalization of the zero divisor graphs to k-zero divisor hyper graphs and list some of the new research directions.
- To examine new zero divisor graph is complete, planar, connected etc.
- To determine Is new generalised graph colourable or not.?

5.Methodology

Methodology will includes the following steps

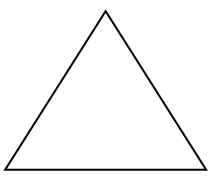
1. To refer the relevant references books on graph theory and zero divisor graphs.
2. To refer and study basic definition of zero divisor graphs.
3. To study and obtain zero divisor graph of some ring by using previous research.
4. To study the past research on zero divisor graph and use it to obtain new appropriate result.
5. To carry out survey for latest development of zero divisor.
6. To obtain zero divisor graphs of some ring by using new result.
7. To obtain zero divisor graphs of modules.

6.Significance of study

When some ring have complicate structure, we can't easily understand it. But by using zero divisor graph we can easily find or check behaviour of ring, also we can find relation between ring theoretic properties and graph theoretic properties. Consider the example we can easily understand x,y,x+y are zero divisor of ring.

Consider the ring $Z_2[X,Y]/\langle x^2,xy,y^2\rangle$.The element of ring are 0,1,x,y,x+y,x+1,y+1,x+y+1. x+y

$xy \equiv 0 \bmod xy$;



$x(x+y)=x^2+xy\equiv 0 \mod xy;$
$(x+y)y=xy+y^2\equiv 0 \mod xy$ x y

7.Plan Of Research

7.1First Year:

- To refer the relevant references books on graph theory and zero divisor graphs.
- To study different types of ring (commutative and non commutative ring).
- We will study important definition like as vector spaces , subspaces , ring , subring ,grid ,coloring partite graph all are useful to get new result. To carry out survey for latest development of zero divisor.

7.2 Second Year:

- To carry out survey for latest development of zero divisor.
- To study and obtain zero divisor graph of some ring .
- To illustrate ring theoretic properties and graph theoretic properties using different zero divisor graph .

7.3 Third Year :

- To obtain relationship in between zero divisor graph of ring and module.
- Publishing some papers in national and international conferences.

7.4 Fourth Year:

- We intend to publish our work in reputed journal and conferences.
- Submitting the thesis.

8. References.

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Research Student — Research Guide

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